Ground Penetrating Radar Migration with Uncertain Parameters

A. J. Patterson, J. M. Tealby and N. M. Allinson.
Department of Electronics, University of York,
York, U.K. YO1 5DD.
Tel. +44 1904 433221 Fax. +44 1904 433222 E-mail. ajp@glenlivet.york.ac.uk

Abstract -- In this paper the focusing principle of Kirchoff migration is described. This allows the introduction of optical focusing techniques that can be used to focus migration. Three focus measures are described that are useful for optimising migration. Simple optimisation routines are implemented that model uncertainties in the migration parameters. The focus measures are then used as cost functions to be maximised. Results show that these measures are useful in optimising migration when there are uncertainties in the parameters.

INTRODUCTION

The technique of migration is often used with Ground Penetrating Radar (GPR) to image subsurface targets. The method converts the wavefield recorded, by the radar, on the surface to a depth profile of the subsurface. To achieve this the surface wavefield is extrapolated to a wavefield at each depth step. The data at time zero is taken as the reflector strength at that depth. In this manner the depth profile is constructed at the desired depths. This is called the depth extrapolation technique [1].

The resolution of migration has been examined in [2]. However, this gives only the theoretical limit. For real GPR data sets there are uncertainties in the parameters, that produce distortions in the migrated image. Two main sources of distortion are uneven sampling on the surface and an unknown wave velocity in the subsurface. These errors in the parameters can drastically reduce the resolution of the migration algorithm.

FOCUSING PRINCIPLE

Reference [2] introduces the concepts of synthetic focused arrays to analyse the resolution of migration. For the 2D case, with a coincident transmitter-receiver radar, the wavefield is recorded over the surface z = 0, at 2N sampling points with spacing Δx_s. The wavefield recorded by a receiver positioned at the point ($n\Delta x_s$, 0, 0) is given by $u(n\Delta x_s, 0, 0, \omega)$. To extrapolate the wavefield to a depth z, Kirchoff summation can be used. This gives the wavefield that would have been recorded by a set of hypothetical receivers positioned at depth z. The values recorded at time zero give the reflector strength at that depth. Reference [2] gives the Kirchoff summation equation as

$$u(x,0,z,\omega) \approx N_2(\omega)\Delta x_s \sum_{n=-N}^{N} (z/r_x)^{3/2} u(n\Delta x_s, 0,0,\omega) \times \exp\left[jk(x-x_s)^2/z\right] \tag{1}$$

where

$$N_2(\omega) = (1/zV)^{1/2}(\omega/\pi)^{1/2}\exp(-j\pi/4), \quad r_x^2 = (x-x_s)^2 + z^2 \tag{2}$$

and V is the wave velocity and k is the wave number. This shows the technique to be equivalent to a synthetic focused array focused at the point (x, 0, z). The array is also weighted by the factor $(z/r)^{3/2}$ and filtered by $N_2(\omega)$. To perform migration the array is successively focused to each point in the subsurface that is to be imaged, and the summation is performed. Examining (1) we can see that the focusing is effectively performed by a lens focused at the depth point.

Reference [2] shows how this analogy can be used to examine the lateral resolution of the array and also uses the depth of focus to show the degradation of resolution for incorrect velocities. This paper takes the analogy further to provide measures of image focus that can be used in an optimisation procedure. These can be compared with similar focus measures from optical systems

FOCUS MEASURES

The focusing of the lens function in migration is determined by the velocity, surface sampling points and depth. If these are not correct then the focal point of the lens will be incorrectly positioned and the migration will not focus properly. The similarity with an optical lens system allows the use of optical focusing theory to provide measures of focus for a migrated image. Focusing techniques have long been used in optical systems to properly focus an image, e.g. auto-focusing in cameras [3] and deformable mirrors in telescopy to correct for atmospheric distortions [4]. Reference [3] derives and proves several useful focus measures for a simple camera lens system. Due to the similarities between optical focusing and migration these measures are also useful for GPR migration. The three measures used in this work are:

0-7803-2567-2/95 $4.00 © 1995 IEEE

Image energy,

$$M_1(i) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|u_i(x,0,z)\right|^2 dx\,dz, \tag{3}$$

energy of the image gradient,

$$M_2(i) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|\nabla u_i(x,0,z)\right|^2 dx\,dz, \tag{4}$$

and energy of the image Laplacian,

$$M_3(i) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|\nabla^2 u_i(x,0,z)\right|^2 dx\,dz, \tag{5}$$

where i is the iteration number in an optimisation scheme. For the optical system, these measures are known to be sound, monotonic and unimodal.

The measures are applied to the migrated image to determine the image focus. Equation (1) shows that the array wavefield is multiplied by a weighting factor and then filtered. The filter characteristics depend upon the wave velocity, therefore if different velocities are used on the same data set, different amplitude terms will modify the wavefield. To allow for a fair comparison between images these amplitude terms must be normalised before measuring the focus.

OPTIMISATION

The two parameters of migration that are examined in this paper are subsurface velocity and surface sampling positions.

Migration fundamentally depends on the velocity at which the wave travels in the subsurface. It is assumed constant, but it is not known exactly. In practise it can only be estimated within a given error range. This allows a probability function to be given to the velocity. This function can be used to select velocities in an iterative procedure. Here, a Gaussian distribution is used for the velocity which fits well with practical estimates. A Metropolis style algorithm [5] is used to maximise the focus function of the migrated image. An initial estimate is made of the velocity and the data is migrated. The velocity is then updated using a conditional probability function depending on the previous velocity. An update is accepted if it increases the focus value. If the focus is decreased, the update is accepted with probability exp(-Focus Value/ T), where T is called the annealing temperature. In some cases the temperature is kept at zero thus allowing increases in focus value only. This makes the algorithm more vulnerable to getting trapped in local maxima but increases the speed of convergence. However, the algorithm can still escape some local maxima when the temperature is zero due to the random updating of the velocity. It is this latter approach that proved most useful for optimising the velocity.

GPR data is collected by translating the radar over the surface at a constant velocity. The radar collects data samples at constant time intervals, thus the sampling points are evenly spaced on the surface. In practise it is not possible to move the radar at a constant velocity. This results in uneven sampling points that distort the radar image. The error can be modelled by assuming a probability distribution for the sampling points' deviation from a mean. Again, a Gaussian distribution is useful in describing the errors. Also, there are high correlations in the errors for neighbouring samples and this allows a large reduction on the number of parameters to be optimised. The iterative procedure selects a set of sampling points from the probability distribution and migrates the data, measuring the focus. The Metropolis style algorithm updates the parameters for the sampling points in order to maximise the focus, in a similar manner to that described previously.

RESULTS

Fig. 1a and 1b show graphs of the three focus measures for varying velocities (normalised to 1.0) for a single point scatterer and a real GPR data set respectively. They show clearly that the measures have a maximum for the correct velocity (1.0)

Fig. 2a shows how using an incorrect velocity (by 5%) can degrade the lateral resolution. The two point scatterers are not discernible in the migrated image. Fig. 2b shows the migrated image found using the optimisation algorithm. It has a velocity of 1.02, 2% error. This solution was found in 10 iterations with the annealing temperature set to zero.

Fig. 3a shows how uneven surface sampling degrades resolution and introduces false features to a point scatterer. Fig. 3b shows the optimised migrated image. This was found in 16 iterations with the annealing temperature set to zero.

CONCLUSIONS

The results show that optical focus measures are of use in focusing GPR migration. They provide a maximum value for the most focused image and allow for the correction of distortions found in real data sets. The method has the advantage of using the actual field values in creating an image, as opposed to a LMS method that needs to create synthetic data to fit the real data.

REFERENCES

[1] D.Loewenthal, L.Lu, R.Roberson, and J.Sherwood, "The Wave Equation Applied to Migration," Geophysical Prospecting, Vol.24, pp.380-399, 1976.

[2] M.H.Safar, "On the Lateral Resolution Achieved by Kirchoff Migration," Geophysics, Vol.50, N0.7, pp.1091-1099, 1985.

[3] M.Subbarao, T.Choi, A.Nikzad, "Focusing Techniques," Optical Engineering, Vol.32, No.11, pp.2824-2836, 1993.

[4] N.C.Mehta, C.W.Allen, "Dynamic Compensation of Atmospheric Turbulance with Far-Field Optimization," J.Opt.Soc.Am.A, Vol.11, No.1, pp.434-443, 1994.

[5] S.Kirkpatrick, C.D.Jr,Gelatt, M.P.Vecchi, "Optimization by Simulated Annealing," Science, Vol.220, pp.671-680, 1983.

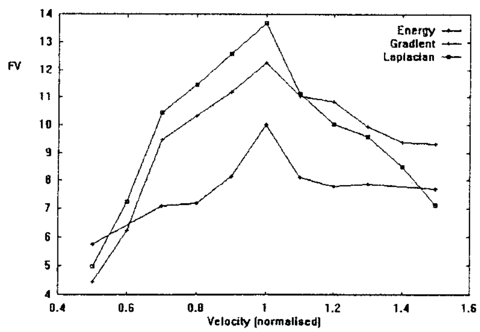

Fig. 1a, Graph of Focus Value against velocity for a point scatterer.

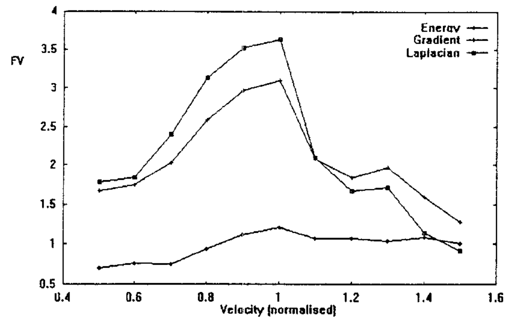

Fig. 1b, Graph of Focus Value against velocity for a real data set.

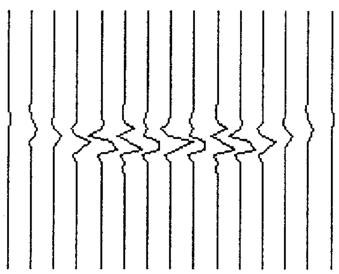

Fig. 2a, Migrated image of two point scatterers with an incorrect velocity of 5%.

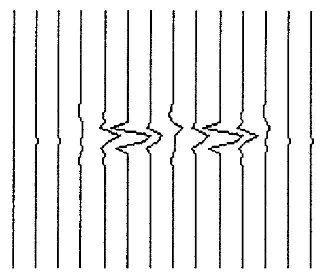

Fig. 2b, Optimised migrated image of two point scatterers. The velocity is in error by 2%.

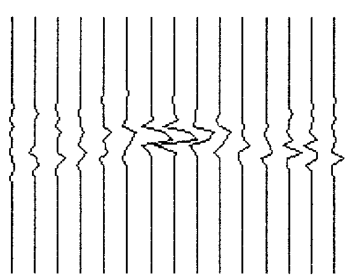

Fig. 3a, Migrated image of a point scatterer with uneven surface sampling.

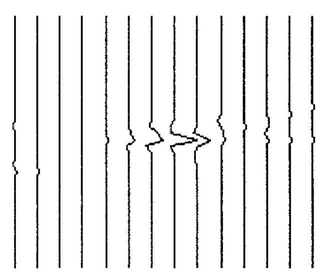

Fig. 3b, Optimised migrated image of a point scatterer with uneven surface sampling.